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ABSTRACT

In near surface electrical exploration, it is often necessary to estimate the Cole-Cole model parameters according to the measured multi-frequency complex resistivity spectrum of ore and rock samples in advance. Parameter estimation is a nonlinear optimization problem, and the common method is least square fitting. The disadvantage of this method is that it relies on initial value and the result is unstable when data is confronted with noise interference. To further improve the accuracy of parameter estimation, this paper applied artificial neural network (ANN) method to the Cole-Cole model estimation. Firstly, a large number of forward models are generated as samples to train the neural network and when the data fitting error is lower than the error threshold, the training ends. The trained neural network is directly used to efficiently estimate the parameters of vast amounts of new data. The efficiency of the artificial neural network is analyzed by using simulated and measured spectral induced polarization data. The results show that artificial neural network method has a faster computing speed and higher accuracy in Cole-Cole model parameter estimation.

INTRODUCTION

Induced polarization (IP) is a physical and chemical phenomenon that occurs when the underground medium is stimulated by external current (Wait, 1959). As most metal-rock ores such as gold, silver, lead, zinc, chromium, copper and iron have strong induced polarization effect, IP method is widely used in mineral exploration and petrophysical survey (Zonge *et al.*, 1975). The resistivity of rock ore specimens containing induced polarization effect is a complex number varying with frequency. The Cole-Cole model can be used to fit the multi-frequency impedance information of various rocks and ores (Cole and Cole, 1941; Pelton *et al.*, 1978). In practical exploration, it is necessary to estimate the Cole-Cole model parameters based on the measured multi-frequency complex resistivity and to infer the material composition and internal structure of the specimen.

At present, scholars have proposed many quasi-linear methods for parameter estimation of Cole-Cole model, including direct inversion method (Xiang *et al.*, 2001), nonlinear least squares method (Freeborn *et al.*, 2012) and least absolute deviation (LAD) method (Yang *et al.*, 2013). These methods are developed from the least-square estimation algorithm and have stronger anti-noise capability. Bayesian inference using Markov-chain Monte Carlo simulation is also widely used for IP parameter estimation (Ghorbani *et al.*, 2007; Chen *et al.*, 2008; Bérubé *et al.*, 2017). This algorithm can obtain the uncertainty of the estimated IP parameters. Some global optimization algorithms are also used for parameter inversion, such as genetic algorithm (Miranda *et al.*, 2008), flower pollination algorithm (FPA), moth-flame optimizer (MFO) (Yousri *et al.*, 2017), and modified particle swarm optimization (MPSO) algorithm (Liu *et al.*, 2018). These approaches are independent of the choice of initial values and also show strong anti-noise abilities. In conclusion, parameter estimation of Cole-Cole model is a highly nonlinear optimization problem. It is a developing trend to reduce the dependence of the algorithm on initial value and improve its anti-interference ability and computing speed. The above methods have been effectively applied in synthetic and laboratory SIP data. However, their computing speed and anti-noise capability still need to be further improved, because noise interference is very complex and the amount of data to be processed is also increasing.

Recently, the development of machine learning algorithms represented by artificial neural network (ANN) provides new ideas for parameter estimation. Compared with the conventional optimization algorithm, machine learning algorithms have stronger nonlinear fitting capability and migration learning ability. They have been widely used in parameter estimation in many fields, such as astrophysics, chemistry, medicine, petroleum science and so on (Dua, 2011; Hatamleh *et al.*, 2015; Sadeghi-Gougharí *et al.*, 2016; George *et al.*, 2018; Wang *et al.*, 2020; Liu *et al.*, 2020; Yan *et al.*, 2020). To improve the computational efficiency, this paper uses the artificial neural network algorithm for Cole-Cole model parameter estimation of laboratory ore and rock samples.

ARTIFICIAL NEURAL NETWORK PARAMETER ESTIMATION

The interpretation of geophysical prospecting results depends on the physical properties of rocks and ores. In laboratory, the complex resistivity spectrum of various specimens is obtained by multi-frequency scanning measuring. We need to estimate the Cole-Cole model parameters to judge the composition and structure of the samples. The Cole-Cole model was first proposed by Cole and Cole (1941), and Pelton *et al.* (1978) firstly introduced it into geophysics. The formula is as follows:

$$\rho(\omega) = \rho_0\left[1 - m\left(\frac{1}{1+(j\omega\tau)^c}\right)\right] \tag{1}$$

where, ρ_0 is resistivity at 0 Hz, m is chargeability, c is frequency correlation coefficient and τ is time constant, which jointly characterize the conductivity and induced polarization effect. In addition, ω is the angular frequency, j is the complex unit, and $\rho(\omega)$ is the calculated complex resistivity.

Artificial neural network is composed of an input layer, multi-hidden layers and an output layer, which can be used to fit nonlinear function by training a large number of groups of input $\mathbf{X}_A$ and output $\mathbf{Y}_A$ samples (Hornik, 1991). For a hidden layer, the output vector $\mathbf{Y}_l$ is the weighted sum of the input vector $\mathbf{X}_l$,

$$\mathbf{Y}_l = f(\mathbf{W}_l\mathbf{X}_l + \varphi_l) \tag{2}$$

where $\mathbf{W}_l$ is the weight coefficient of the hidden layer, f is an activation function, and φ_l is a bias coefficient (Dongare *et al.*, 2012; Gupta, 2013). The output of the current hidden layer is also the input of the next hidden layer. After integrating all the hidden layers, the simulated system output $\mathbf{Y}_S$ is obtained. Finally, the optimal weights $\mathbf{W}_l$ and φ_l of all the hidden layers are solved by back-propagating errors algorithm to minimize the difference C between all the simulated output $\mathbf{Y}_S$ and actual output $\mathbf{Y}_A$ (Rumelhart *et al.*, 1986):

$$C = \sum_{i=1}^{N}\frac{1}{2}\left\|\mathbf{Y}_A\{i\} - \mathbf{Y}_S\{i\}\right\|^2 \tag{3}$$

where, N is the number of the training samples. Then, other input and output samples are used to test the network system. When the testing error is lower than error tolerance, the neural network system can be used to predict new output. The Cole-Cole model parameter estimation algorithm based on ANN mainly includes three steps: First, a large number (usually thousands) of Cole-Cole model parameters are randomly generated, and then the multi-frequency complex resistivity spectrum are calculated by using equation 1 at given frequencies. Secondly, the multi-frequency complex resistivity and the corresponding Cole-Cole model parameters are taken as input and output samples respectively to train a neural network model. Finally, the trained optimal neural network system after testing is directly used to predict new output according to corresponding new input data. Figure 1 shows the schematic diagram of Cole-Cole model parameter estimation based on artificial neural network.

Additionally, the training effect of artificial neural network is further improved by adopting cross-validation, adding random disturbances, and data normalization. Firstly, during the training, all samples are randomly divided into training group, testing group and cross-validation group by 70%, 15% and 15%. The net is to try to use different training sets and validation sets to conduct multiple groups of different training and validation for the model to prevent overfitting. Secondly, we add about 3% random disturbance to the input samples, and then conduct training to enhance the stability and anti-interference ability of the network. Thirdly, we take logarithm of the input and out samples for normalization because that the value range of the model parameters vary widely.

SIMULATED COMPLEX RESISTIVITY SPECTRUM DATA TESTING

Firstly, we test the ANN algorithm by using simulated data. 10,000 groups of Cole-Cole model parameters are randomly generated, where the range of zero-frequency resistivity is 0.01~10,000 ohm-m, the value range of chargeability is 0~1, the range of frequency correlation coefficient is 0~1, and the range of time constant is 0.0001~10,000 s. The frequency range is 2^{-10}~2^{10} Hz, and then the amplitude and phase of multi-frequency complex resistivity are calculated by using the formula (1). Then, 5,000 sets of models are used to train neural networks, and another 5,000 sets of models are used to test the inversion accuracy. Considering that the

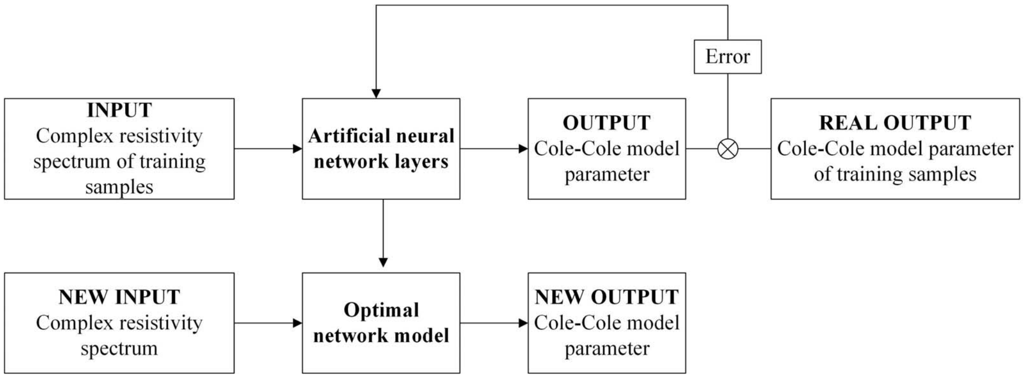

Figure 1 A schematic diagram of Cole-Cole model parameter estimation based on artificial neural network.

amplitude of complex resistivity is sensitive to the zero-frequency resistivity parameter (ρ_0) in the Cole-Cole model, and the phase of complex resistivity is sensitive to the induced polarization parameters (m, c, τ), we train two networks to estimate the resistivity and induced polarization parameters separately.

The neural network consists of three hidden layers with 12 units in each layer. Levenberg-Marquardt algorithm is used for iterative optimization. Finally, after about 120 iterations, the optimal neural networks are obtained, the total time is about 13 s. Then, another 5,000 groups of test data are used for testing. Firstly, we directly use the test data without noise interference to verify the obtained network, and then, we add 3% noise interference to the data to test the network. Figure 2 shows the comparison of estimated

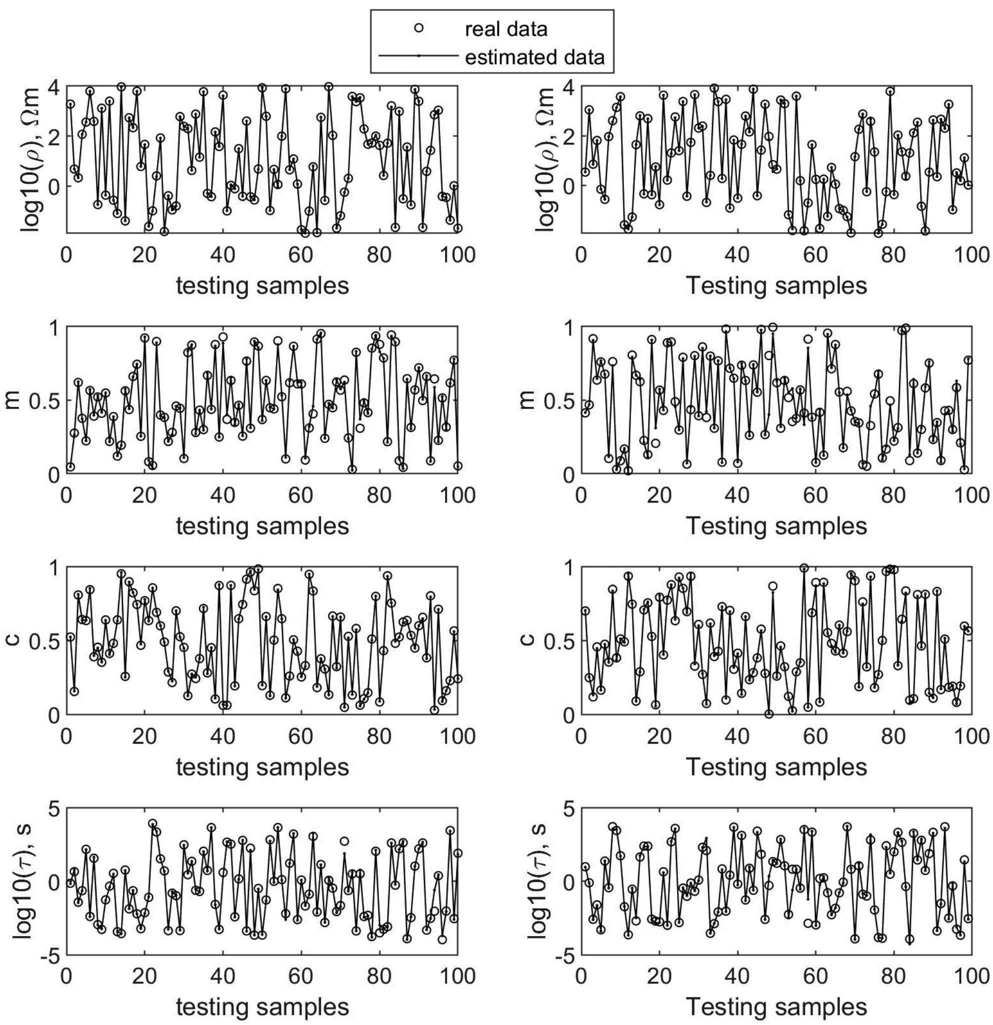

Figure 2 Comparison between the estimated and real model parameters of 100 testing samples; Left: the input data do not contain noise interference; Right: the input data are interfered by 3% random noise.

Table 1 Computation time and percentage of low-error data for artificial neural network (ANN) and least square (LS) methods.

Method	log10(ρ)_error 0.25 (%)	m_error 0.05 (%)	c_error 0.05 (%)	log10(τ)_error 0.25 (%)	Time cost (s)
ANN (without noise)	99.52	94.98	99.86	91.58	0.21
LS (without noise)	99.76	95.76	99.32	94.02	173.25
ANN (with 3% noise)	99.36	92.80	99.86	89.30	0.21
LS (with 3% noise)	93.50	86.42	89.18	80.62	190.54

parameters and real parameters of 100 randomly selected data sets.

To compare the calculation accuracy and time, the least squares algorithm is also used to estimate the Cole-Cole model parameters of these 5,000 test data with and without adding random noise respectively. We then calculate the percentage of low-error data respectively. Low-error means that the absolute error of the logarithmic resistivity is less than 0.25, the absolute error of the chargeability is less than 0.05, the absolute error of the frequency correlation coefficient is less than 0.05, and the absolute error of the logarithmic time constant is less than 0.25. Table 1 shows the error statistics and the calculation time of the two methods. All the calculations are done on an Intel core i5 processor at 2.50 GHz. Due to that the training network can be directly applied in testing data, the inversion takes less than a second. When the testing samples contain no noise interference, the accuracy of ANN is comparable to that of the least-square method. However, when the testing sample contains noise interference, the accuracy of ANN algorithm is higher than that of the least squares method. For both methods, the estimation accuracy of resistivity and frequency correlation coefficient is higher than that of frequency correlation coefficient and time constant.

PRACTICAL COMPLEX RESISTIVITY SPECTRUM DATA PROCESSING

The ANN is further used to estimate the Cole-Cole model parameters from multifrequency complex resistivity spectrum of various kinds of ore-rock samples collected in a mine area, Tibet, China. Figure 3 shows the physical pictures of ore and rock specimens. Specimens No.1 is large-grain pyrite with purity of 30%, other compositions are mainly carbonaceous slate, feldspar and impurities. Specimens No.2 and specimens No.3 are dense massive pyrite with purity of 70% and 90% respectively, other components are calcite and feldspar. Specimens No.3 is also dense massive pyrite with purity of 90%, Other components are feldspar and impurities. Specimens NO.4, specimens NO.5 and specimens NO.6 are galena samples with purity of 60%, 80% and 90% respectively, other components are calcite and feldspar. Specimen NO.7 is sphalerite with purity of 50%, embedded with massive and fine pyrite, with a small amount of carbonaceous slate and feldspar. Specimens No.8 and specimen NO.9 are carbonaceous slate samples with purity of 80% and 90% respectively, with a small amount of veined pyrite embedded. Specimen NO.10 is magnetite with a purity of 90%, other components are some impurities. The estimated parameters are shown in Table 2. We also calculate the multi-frequency complex resistivity according to the estimated parameters and compare them with the observed data in Fig. 4. The processing results of the practical data show that ANN algorithm can effectively identify different types of complex resistivity curves and estimate the parameters of Cole-Cole model.

Table 2 Cole-Cole model parameters estimated based on multi-frequency complex resistivity of ten conductive rock ore samples.

Specimen number	Log10(ρ)	m	c	Log10(τ)
No.01	2.5612	0.1727	0.4459	0.0601
No.02	2.5260	0.6407	0.4005	0.2581
No.03	2.7713	0.6001	0.4053	1.1987
No.04	1.6493	0.3106	0.6233	0.3011
No.05	1.8680	0.5778	0.4878	0.9931
No.06	1.6234	0.2164	0.5645	0.9274
No.07	2.4373	0.6272	0.2821	1.5013
No.08	2.3334	0.7906	0.3529	3.1141
No.09	2.7664	0.8960	0.3191	3.0795
No.10	1.6452	0.3185	0.5600	-0.5213

CONCLUSION

In this paper, we developed an artificial neural network algorithm for the Cole-Cole model parameter estimation of various ore and rock samples. The testing results of simulated data show that the new algorithm has higher accuracy, when measured data is contaminated by noises. We also showed how this method was efficiently used for the parameter estimation of various ore and rock samples collected in a mine area, in Tibet, China. The estimated frequency correlation coefficients and time constants are distinguishable for these specimens. The research in this paper shows that artificial neural network is helpful to improve the accuracy, speed and automation level of rock and ore spectrum parameter estimation.

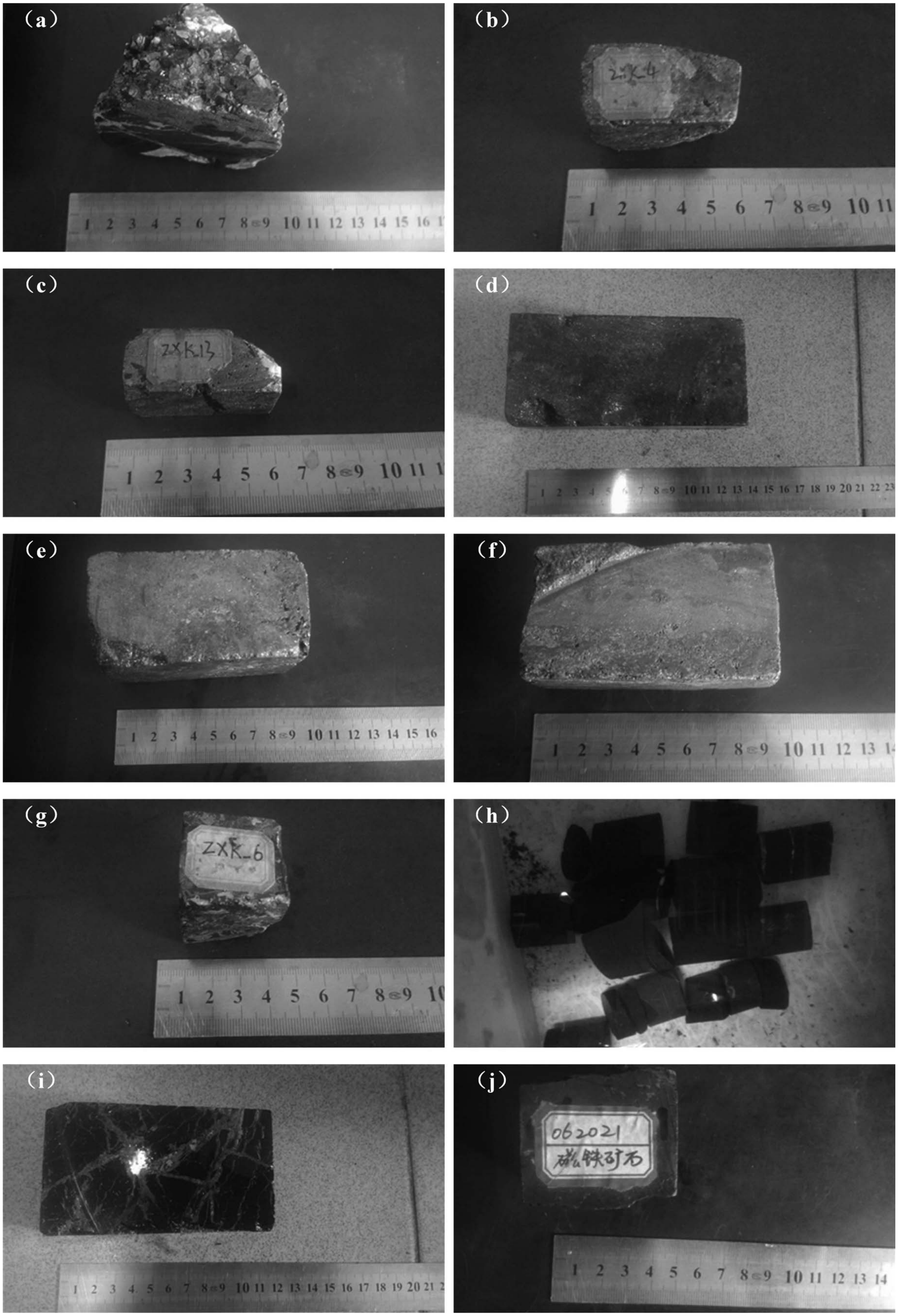

Figure 3 Physical pictures of ore and rock specimens: a) Large-grain pyrite with 30% content; b) Pyrite specimen with 70% content; c) Pyrite specimen with 90% content; d) Galena specimen with 60% content; e) Galena specimen with 80% content; f) Galena specimen with 90% content; g) Sphalerite specimen with 5% content; h) Carbonaceous slate samples with 80% content; i) Carbonaceous slate samples with 90% content; and j) Magnetite with 90% content.

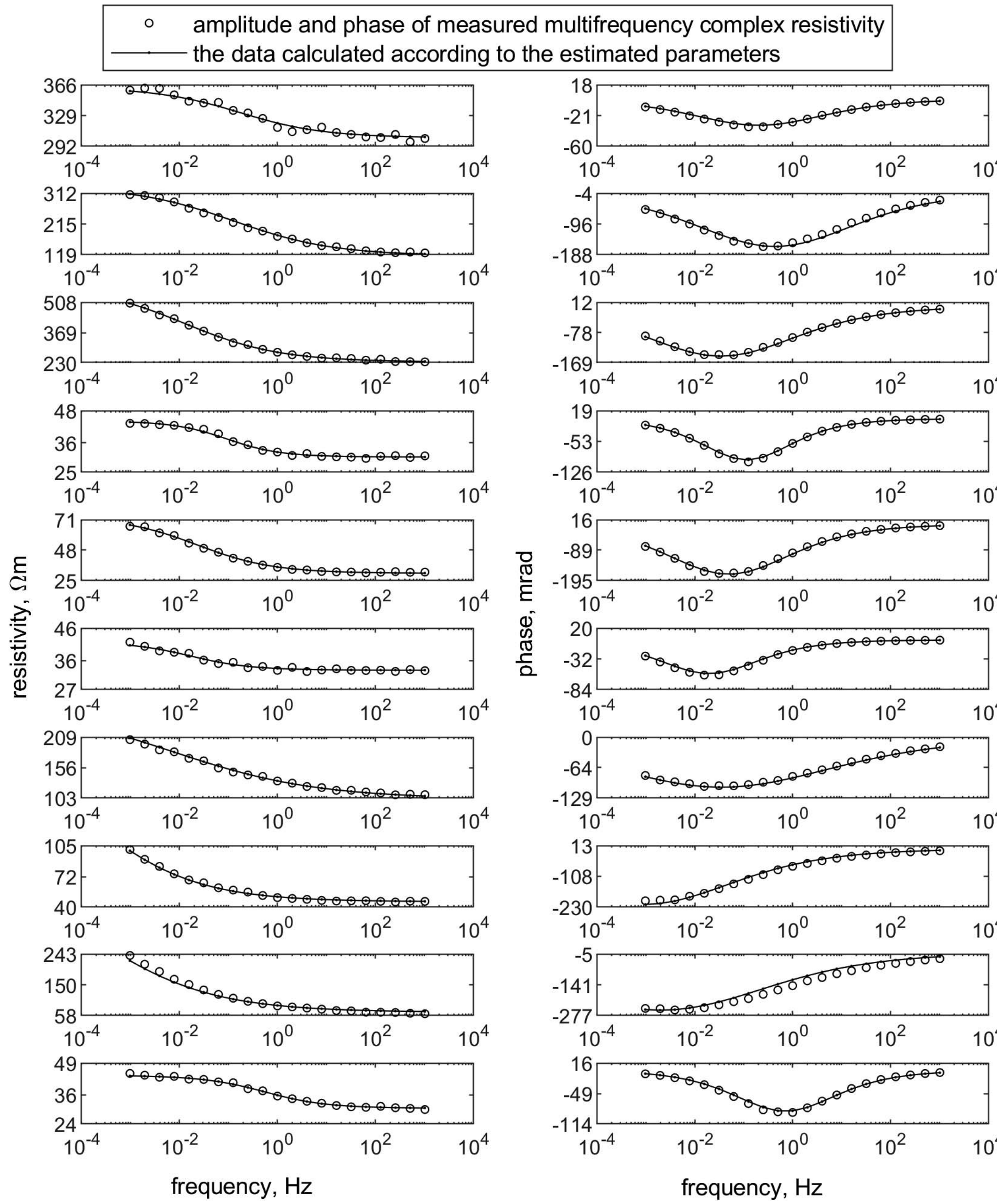

Figure 4 The measured and fitted amplitudes and phases of multi-frequency complex resistivity of 10 conductive rock ore samples; Left: the measured and fitted multi-frequency resistivity amplitude; Right: the measured and fitted multi-frequency phase.

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